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LABORATORY REPORT

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Technical Report No. 30 November 1965

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Research sponsored by the
U. S. Office of Naval Research
Contract Nonr 220(50)

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Received 12 November 1965

The aim of this letter is to show that the relativistic mass effect in a cold plasma in an inhomogeneous magnetic field gives rise to an echolike phenomenon [1] with all the characteristics of the experimentally observed echoes reported by Hill and Kaplan [2].

We consider a plasma in a slightly inhomogeneous magnetic field excited by two short pulses at the electron gyrofrequency, the time separation of the pulses being τ. We ask for the radiation after the second pulse. We treat the problem in the single particle approach.

The radiated intensity depends on the phase correlations between the different particles. More precisely, the time variation of the radiation is given by

$$\Phi(t) = \Big(\sum_{l=1}^{N} r_l\Big)^{-2} \left|\sum_{l=1}^{N} r_l \exp\{\mathrm{i}[\omega_c^l t + \alpha_l]\}\right|^2 , \tag{1}$$

where N is the number of particles considered, r_l is the Larmor radius of the lth particle, ω_c^l is its gyrofrequency, and α_l its phase at time $t = 0$. $\Phi(t)$ is normalized such that it is unity when all particles have the same phase. From (1) one sees that it is sufficient to determine phase *differences*. For this it is convenient to consider the phases in a coordinate system rotating with the average gyrofrequency $\overline{\omega}_c$.

We neglect the initial energy of the electrons (cold plasma approximation). At the end of the first pulse all particles have in this approximation the same energy and the same phase. We now account for the field inhomogeneities by attributing a different gyrofrequency to each electron. (This implies that the inhomogeneities are perpendicular to the field lines.) We assume a distribution $g(\eta)$ over the different gyrofrequencies, where $\eta = \Delta_{\mathrm{inh}}\omega_c$ is the deviation from the average gyrofrequency due to the inhomogeneities. The relative phase of a particle at a time τ after the first pulse is

$$\varphi(\eta_l, \tau) = \eta_l \tau . \tag{2}$$

After the second pulse the particles have *different* energies according to their phase at the onset of this pulse. This give rise to additional differences in the gyrofrequencies due to the relativistic mass effect. With $v^2 \ll c^2$ we have after the second pulse

$$\Delta^*\omega_c^l = A(B - r_l^{*2}/R^2) + \eta_l \tag{3}$$

with $A = \overline{\omega}_c^3 R^2/2c^2$, R being the Larmor radius after the first pulse and r_l^* that after the second pulse. B is an arbitrary constant which defines the particle with respect to which $\Delta^*\omega_c$ is measured. The relative phase of a particle at time t, now measured from the second pulse, is then given by

$$\varphi_l(t) = \varphi_l^*(t=0) + \Delta^*\omega_c^l t \tag{4}$$

$\varphi_l^*(t=0)$ and r_l^* of a particle depend on its phase at the onset of the second pulse and are given by

$$r_l^{*2}/R^2 = 1 + D^2 + 2D\cos[\varphi(\eta_l, \tau) - \varphi_0] \tag{5}$$

$$\varphi_l^*(t=0) = \varphi_0 + \beta_l \tag{6}$$

‡ This work was sponsored by the U.S.Navy, Office of Naval Research.
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$$\sin \beta_l = (R/r_l^*) \sin [\varphi(\eta_l, \tau) - \varphi_0] , \tag{7}$$

where D gives the strength of the second pulse relative to the first and φ_0 the phase of the electric field of the second pulse relative to a particle with $\varphi(\tau) = 0$. With these relations we have, after the second pulse:

$$\Phi(t) \sim \left| \int \mathrm{d}\eta \; g(\eta) \{D \exp[\mathrm{i}f(\eta)] + \exp[\mathrm{i}f_1(\eta)]\} \right|^2 \tag{8}$$

$$f(\eta) = \varphi_0 + At[B - 1 - D^2 - 2D \cos(\eta\tau - \varphi_0)] + \eta t \tag{9a}$$

$$f_1(\eta) = \eta\tau - \varphi_0 + f(\eta) . \tag{9b}$$

As there is a cosine in the exponential functions, we make an expansion into Bessel functions:

$$\exp[\mathrm{i}f(\eta)] = \sum_{k=-\infty}^{+\infty} J_k(2ADt) \exp\{\mathrm{i}[\eta(t + k\tau) - (k-1)\varphi_0 - \tfrac{1}{2}k\pi + \psi]\} . \tag{10}$$

ψ stands for all terms not depending either on η or the index of the Bessel functions. From (10) it follows that there are maxima of $\Phi(t)$ at times

$$t_m = m\tau , \qquad m = 1, 2, 3, \ldots \tag{11}$$

$$\Phi(t_m) \approx (1 + D^2)^{-1} \{ |DJ_m(2ADt_m|^2 + |J_{m-1}(2ADt_m)|^2\} . \tag{12}$$

If the argument of the Bessel functions is large we have $\Phi(t_m) \sim 1/t_m$. When collisions are taken into account (12) is to be multiplied by an exponential function. The shape of the radiation peaks is the square of the Fourier transform of $g(\eta)$.

As an example of their experimental results Hill and Kaplan [2] give an oscillogram showing two radiation maxima at times τ and 2τ, the intensity ratio being 8 : 1. This ratio is essentially determined by collisions, as τ is of the order of the reported decay constant.

In the case of three exciting pulses Hill and Kaplan find an echo at a time τ after the third pulse. The dependence of its intensity on T (time between the second and the third pulse) turns out to be determined by the inelastic collisions only.

If one treats the three-pulse case at first in the collisionless approximation analogous to the two-pulse case, one obtains an expression for $\Phi(t)$ now consisting of sums over products of Bessel functions multiplied by an exponential function of the kind

$$\exp\{\mathrm{i}[\eta(t + (k+m+n)\tau + (l+m)T + \psi_m]\} , \tag{13}$$

where k, l, m, n are summation indices. From this one concludes that radiation maxima arise at times

$$t_{KL} = K \;\; + LT , \qquad \pm K, L = 0, 1, 2, \ldots \tag{14}$$

t now being measured from the third pulse. If one now assumes that the phases of the electrons are, due to elastic collisions, randomized between the second and third pulse, but the individual energies are preserved, one must consider the phase ηT in (13) as a statistical quantity over which one must integrate. This integration leads to the cancellation of all terms with $L \neq 0$, while those with $L = 0$ survive.

The essential difference with the two-pulse case is that, after the second pulse, there is information stored not only in the phases, but also in the energy distribution, which is not destroyed by elastic collisions.

If the plasma dimensions are of the order of the wavelength (or larger), the k-vectors of the exciting pulses have to be perpendicular to the magnetic field. Otherwise, particles excited at different phases can exchange their places and spoil by that the generated phase correlations. If the initial temperature of the plasma is too high, the radiation maxima vanish [3].

References

1. R.W.Gould, Physics Letters 19 (1965) 477.
2. R.M.Hill and D.E.Kaplan, Phys.Rev.Letters 14 (1965) 1062.
3. W.H.Kegel, to be published.

UNCLASSIFIED

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY (Corporate author) California Institute of Technology Pasadena, California	2a. REPORT SECURITY CLASSIFICATION 2b. GROUP

3. REPORT TITLE

ON THE THEORY OF PULSE STIMULATED RADIATION FROM A PLASMA

4. DESCRIPTIVE NOTES (Type of report and inclusive dates)

Technical Report, December 1965

5. AUTHOR(S) (Last name, first name, initial)

KEGEL, Wilhelm H.
GOULD, Roy W.

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 15, 1965	2	3

8a. CONTRACT OR GRANT NO. Nonr 220(50) b. PROJECT NO. c. d.	9a. ORIGINATOR'S REPORT NUMBER(S) Technical Report No. 30 9b. OTHER REPORT NO(S) (Any other numbers that may be assigned this report)

10. AVAILABILITY/LIMITATION NOTICES

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11. SUPPLEMENTARY NOTES	12. SPONSORING MILITARY ACTIVITY

13. ABSTRACT

By including the relativistic mass change in the motion of electrons gyrating in a slightly inhomogeneous field, it is possible to account for the cyclotron echoes observed by Hill and Kaplan.

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Security Classification

14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
CYCLOTRON RADIATION						
PLASMA RADIATION						
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